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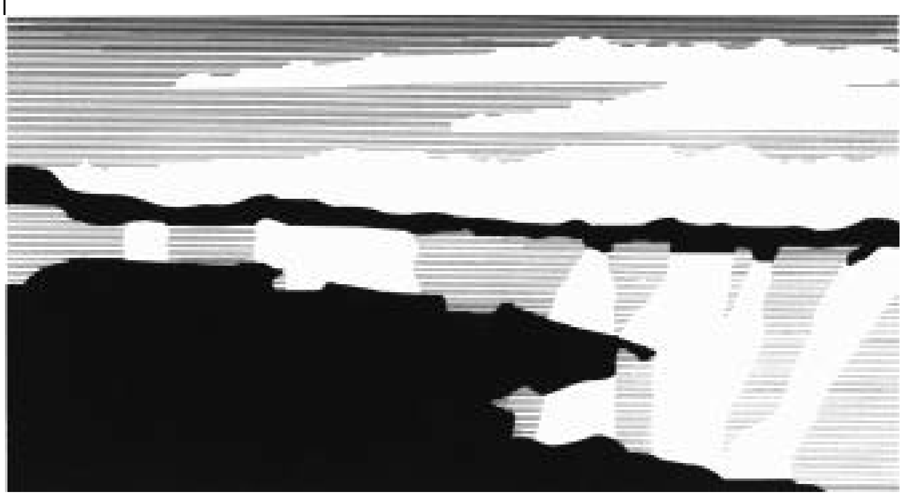

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THE INSTANTON LIQUID MODEL OF QCD

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Within a microscopic model for the non-perturbative vacuum of QCD, hadronic correlation functions are calculated. In the model the vacuum is a statistical, interacting ensemble of instantons and anti-instantons at the scale of Λ_{QCD}. Hadronic two-point as well as three-point correlation functions are evaluated and compared with phenomenological information about the spectra, couplings and form factors. Especially the electro magnetic form factor of the pion is obtained and new predictions for the charm contribution to DIS structure functions are made.

1 The instanton liquid model

The theory of the strong interactions, QCD, is usually divided into two regimes. The short distance phenomena, which are accessible by perturbative calculations order by order in the coupling constant, and the medium and large distance physics, which is non-perturbative and not directly accessible, except for numerical studies of QCD on a lattice (e.g. Ref.[1]). However in recent years our knowledge about the vacuum structure of QCD has been improved and lattice QCD calculations were able to show that the earlier speculations about an instanton dominated ground state of QCD were not only qualitatively right but also quantitatively [1]. The instanton liquid model itself however was proposed already more than a decade ago by Shuryak [2] (or cf. Ref.[3] for a more popular version) and Diakonov and Petrov [4] and contained most of the nowadays known physics already.

The starting point of the instanton liquid models is a semiclassical expansion of the QCD partition function around the classical solutions of the Euclidean Yang-Mills equations. These so-called BPST instantons [5] in singular gauge are given by

$$A_\mu^a(x) = \frac{2\bar{\eta}_{a\mu\nu} x_\nu \rho^2}{x^2(x^2+\rho^2)} \qquad (1)$$

Here the $\bar{\eta}_{a\mu\nu}$ are the antisymmetric 't Hooft symbols [6] and ρ is a scale parameter, the instanton size, which however does not appear in the action of an instanton, since the theory is classically scale invariant. In an ensemble of instantons - and this is what is needed in order to describe the gross properties of the vacuum such as spontaneous chiral symmetry breaking - one has also

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arbitrary positions z_I and color orientations $U_I \lambda^a U_I^\dagger$, so that the field strength for the instanton I can be written as

$$G^I_{\mu\nu}(x) = -\frac{4\rho^2}{((x-z_I)+\rho_I^2)^4}\left(\bar\eta_{a\mu\nu} - 2\bar\eta_{a\mu\alpha}\frac{x_\nu x_\alpha}{x^2} - 2\bar\eta_{a\alpha\nu}\frac{x_\mu x_\alpha}{x^2}\right) U_I \lambda^a U_I^\dagger \quad (2)$$

In the presence of these instantons the Dirac operator has a zero mode, $\not{D}\Psi_0(x) = 0$. It turns out that the corresponding zero mode propagator, $\sim \Psi_0(x)\Psi_0^\dagger(y)$, is left handed for instantons and right handed for anti-instantons. Now comes the second important observation. The singlet axial current in QCD is not conserved but the divergence given by the famous ABJ anomaly

$$\partial_\mu j_{5,\mu}(x) = \frac{N_f}{16\pi^2} G^a_{\mu\nu} G^a_{\mu\nu} + 2m_f \sum_f \bar q_f i \gamma_5 q_f \quad (3)$$

As a result of the instanton zero modes, one can show that the change in the axial charge in this vacuum, $Q_5(t=-\infty) - Q_5(t=\infty)$, is given by the difference $2N_f(n_L - n_R)$ of left and right handed zero modes and therefore only the zero modes account for the non-conservation of the axial charge. This was also one of the first hints that instantons could provide the mass for the η' meson, which otherwise would be a Goldstone boson.

The instanton liquid model is then defined by the partition function for an ensemble of instantons and anti-instantons

$$Z = \sum_{N^+,N^-} \frac{1}{N^+!N^-!} \int \prod_i^{N^+ + N^-} [\mathrm{d}\Omega_i n(\rho_i)] \exp\left(-S_{int}\right) \prod_f^{N_f} \det\left(\not\partial - i\not{A} - im_f\right) \quad (4)$$

where $\mathrm{d}\Omega_i$ accounts for the whole set of collective coordinates and $n(\rho_i)$ is the semiclassical size distribution. Note that the functional integral over the gauge fields is reduced to an ordinary integral over the collective coordinates. The numerical solution of eq. (4) was done first in Ref.[7] and the result can be seen in Fig. 1 where the topological charge is shown on a slice (x,y) of a rectangular, four dimensional space time box with dimensions $(5.7\text{fm})^2(2.8\text{fm})^2$. There are 256 instantons and anti-instantons in the box, resulting in an average instanton density of $\bar n = 1\text{fm}^{-4}$. In the simulation the average size of the instantons comes out to be $0.35 - 0.4$ fm .

2 The point to point correlation function

A methodical tool to study hadron properties is the point to point correlation function of the type

$$\Pi_{a,b}(x) = \langle 0 \mid j_a(-x/2) j_b(x/2) \mid 0 \rangle \quad (5)$$

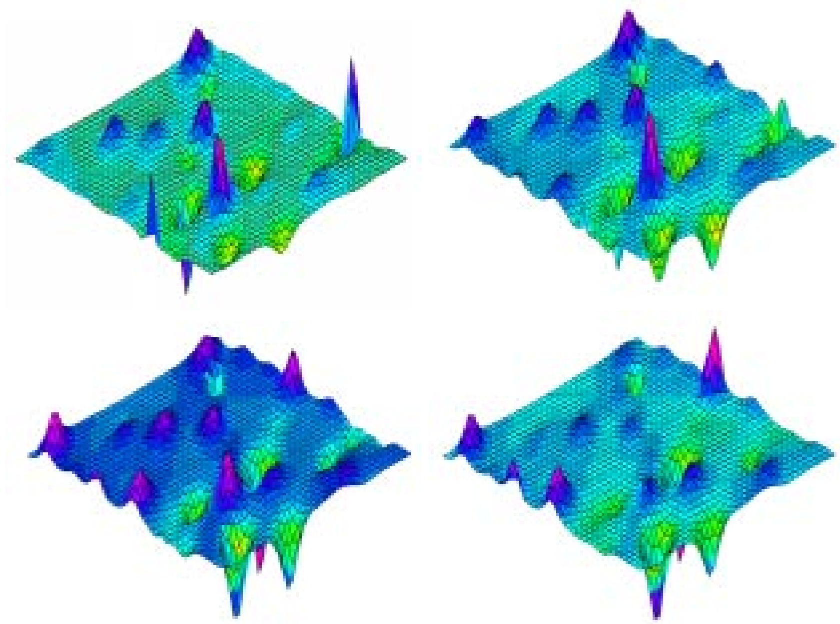

Figure 1: The topological charge density on a (x,y) slice of a four dimensional $(5.673)^2 \times (2.82)^2$ fm^4 box for different values of the Euclidean time $\tau = 0.0, 0.1, 0.2, 0.3$ fm.

where the $j_a(x)$ are the operators corresponding to any kind of hadron, either mesons or baryons in the case of pure quark states, or gluon fields in the case of glueballs or hybrid states of quark and gluons. One advantage of using the correlation function is that it contains small and large distance physics together. For very small distances one can use the free field propagators in (5) complemented with perturbative, radiative corrections. If the quarks or gluons propagate to larger distances, they start more strongly to interact with the vacuum fields and in this regime one can use the operator product expansion (OPE). As distances become large, say $x \simeq 0.3$fm, the interaction becomes quite complicated and one has to use either numerical lattice calculations or some model of the vacuum, e.g. the instanton liquid model.

But not only relates the large distance behaviour of the correlation functions (5) to the hadron spectra and couplings, in the case of vector currents the imaginary part of the spectral density $\mathrm{Im}\Pi_\rho(s)$ is proportional to experimentally known cross sections for e^+, e^- annihilation into hadrons [8]. Specifically in the ρ channel one has $\mathrm{Im}\Pi_\rho(s) = (1/6\pi)R_\rho(s)$ where the cross section is

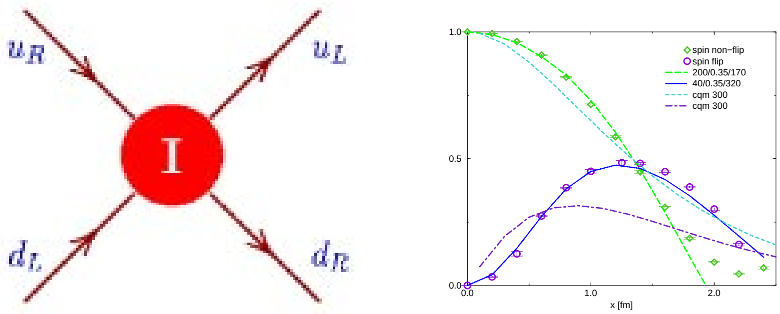

Figure 2: The LHS shows an instanton induced quark vertex for 2 flavors. The RHS shows the spin flip and spin non-flip amplitudes of a quark propagator normalized to the free field propagator. The fitting curves correspond to quark propagators with dynamical, momentum dependent quark masses. Best fits to constant quark mass propagators are also shown.

normalized according to

$$R_\rho(s) = \frac{\sigma_{e^+e^-\to\rho}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)} \tag{6}$$

This means the correlation functions actually contain the whole information about the spectral density and therefore the spectrum of QCD in a given channel and not only the lowest lying resonances. This clearly outperforms conventional approaches like the Nambu–Jona-Lasinio model [9] and to which the instanton liquid model is related only in the large Euclidean time limit and for small instanton density.

3 The pion

The lowest excitations of the QCD vacuum are the Goldstone bosons of the spontaneously broken chiral symmetry, the pseudoscalar, isovector pions. From their correlation function

$$\begin{aligned} \Pi(-x/2, x/2) &= \langle 0 \mid \mathcal{T}\{j_5(-x/2) j_5^\dagger(x/2) \mid 0\rangle \\ &= \langle \mathrm{Tr}\ S_u(-x/2, x/2)\gamma_5 S_d(x/2, -x/2)\gamma_5\rangle \end{aligned} \tag{7}$$

with $j_5(x) = \bar{q}(x) i\tau^\pm \gamma_5 q(x)$, one can see that the quark zero mode progagator for a single instanton would give a contribution to the correlator. The reason for this is the chirality structure of the pion. In a more quantitative way, in Fig. 3 the pion correlator normalized to the free field correlator is shown and the fit with the phenomenological form for the spectral density agrees with the experimental data. In conclusion the phenomenologically wanted, strong enhancement of the correlator is highly non-trivial and entirely due to the zero modes. On the other hand QCD sum rule approaches are known to fail in this channel.

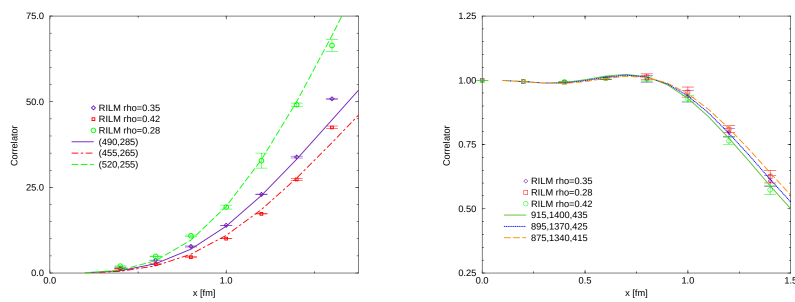

Figure 3: The pion two point correlator (left) and the rho meson correlator (right) for the random ensemble. Different instanton sizes are used and the phenomenological fit parameters (coupling constant, mass for the pion abd mass, coupling constant, continuum threshold for the rho) are given in the legend. The numerical pion mass is for $m_u = 20$MeV and has to be rescaled to the physical current masses.

4 The rho meson

In the case of the ρ, the correlator is given by

$$\begin{aligned} \Pi_{\mu\nu}(-x/2, x/2) &= \langle 0 \mid \mathcal{T}\{j_\mu(-x/2) j_\nu^\dagger(x/2) \mid 0\rangle \\ &= \langle \mathrm{Tr}\ S_u(-x/2, x/2)\gamma_\mu S_d(x/2, -x/2)\gamma_\nu\rangle \end{aligned} \tag{8}$$

and comparison with eq. (7) shows that this time the zero modes can contribute only if one of the propagators is a right handed one and the other a left handed one. In other words, one needs an instanton and an anti-instanton at the 'same time', which makes the correlator weaker and explains qualitatively the higher mass of the ρ (cf. Fig. 3). In the simulation we obtain[10] $m_\rho \simeq 900$MeV and a threshold at 1400MeV, representing the ρ' and higher resonances. Details of other two-point correlation functions can be found in Ref. [11].

5 The electro magnetic formfactor

The interaction of the hadrons with external fields is described by three-point correlation functions of the form

$$\Pi_\mu(-x/2, x/2, y) = \langle 0 \mid \mathcal{T}\{j_5(-x/2) j_{Q,\mu}(y) {j^\dagger}_5(x/2)\} \mid 0 \rangle \tag{9}$$

where $j_{Q,\mu}(y)$ is the electro magnetic current. The spectral density in this case contains now in addition the information for the formfactor and therefore also the EM pion squared radius. The results of the numerical simulation [10] in Fig. 4 shows that the pion size is strongly dependent on the instanton size and the experimental pion size corresponds to the mean instanton size of $0.35-0.4$fm. Future high precision experiments of the EM form factor at TJNAF[12] will further increase our knowledge about the instanton size.

6 Charm in DIS structure function

Since the instantons are a natural source to saturate the matrix elements of eq. (3) in a nucleon state, it is useful to study the spin structure of the hadrons in the instanton liquid via the three-point correlator

$$(\Pi_\mu)_{\alpha\beta}(-x/2, x/2, y) = \langle 0 \mid \mathcal{T}\{\eta_\alpha(-x/2) j_{\mu,5}(y) {\eta^\dagger}_\beta(x/2)\} \mid 0 \rangle \tag{10}$$

where $\eta_\alpha(-x/2)$ is an operator with Dirac index α and $j_{\mu,5}(y)$ is the singlet axial vector current. Now in deep inelastic scattering (DIS) the spin structure function of the proton is given by

$$\Gamma_1^p(Q^2) = C_{NS}(Q^2)\left(\frac{g_A^3}{12} + \frac{g_A^8}{36}\right) + \frac{1}{9} C_S(Q^2)\,(\Sigma + 2\Delta c) \tag{11}$$

where the $g_A^{(3,8)}$ are the axial vector couplings from nucleon beta and hyperon decays. The $C_{NS}(Q^2), C_S(Q^2)$ are the coefficients containing the perturbative corrections. and $g_A^{(0)} = \Delta\Sigma + 2\Delta c$ is the measureable singlet axial coupling constant. It is related to the quark spin contribution to the proton spin and the centerpoint of the proton 'spin crisis'[13]. Since conventional quark models give a $\Delta\Sigma$ which is much bigger ($\simeq 0.6-0.7$) than the experimental findings ($\simeq 0.33$, cf. Ref. [14] for a recent update) and other approaches seem to indicate a very small Σ [15,16] much controversy is about the remaining spin. One suggestion by Halperin and Zhitnitsky[17] was that the non-perturbative intrinsic charm component of the proton, definded by $s_\mu \Delta c = \langle p, s \mid (\bar{c}\gamma_\mu\gamma_5 c)_{\mu^2} \mid p, s\rangle$ might resolve the puzzle. To this aim one performs an expansion of eq. (3) for a heavy quark and finds [17]

$$-\partial_\mu(\bar{c}_E\gamma_\mu\gamma_5 c_E) = +i\frac{1}{2^5 3\pi^2 m_c^2} f^{abc} G^a_{\mu\nu}\tilde{G}^b_{\nu\alpha}G^c_{\alpha\nu} + \mathcal{O}(G^4/m_c^4) \qquad (12)$$

Note that the anomaly term of eq. (3) has disappeared from the divergence as it is expected since in the heavy-quark limit there should be no contribution from quarks, which are not valence quarks. In the instanton background the $G\tilde{G}G$ term evaluates to

$$-\frac{1}{2^5 3\pi^2 m_c^2} f^{abc} G^a_{\mu\nu}\tilde{G}^b_{\nu\alpha}G^c_{\alpha\nu} = Q_I \frac{16}{\pi^2}\frac{1}{m_c^2}\frac{\rho^6}{(\rho^2+(y-z_I)^2)^6} \qquad (13)$$

and is therefore more strongly localized than the anomaly term

$$\frac{1}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} = Q_I \frac{1}{\pi^2}\frac{12\rho^4}{(\rho^2+(y-z_I)^2)^4} \qquad (14)$$

itself. In the limit of three massless quark flavours and a fourth heavy quark, the ratio $\Delta c/\Delta\Sigma$ is simply proportional to a nucleon correlator with $G\tilde{G}$ and $G\tilde{G}G$ as operators. In Fig. 4 we have plotted the correlator for the ratio of $< G\tilde{G} > / < G\tilde{G}G >$ and found [18] the values $\Delta c/\Delta\Sigma = -0.20 + 0.04$ for the random ensemble and $\Delta c/\Delta\Sigma = -0.08 + 0.006$ for the interacting ensemble.

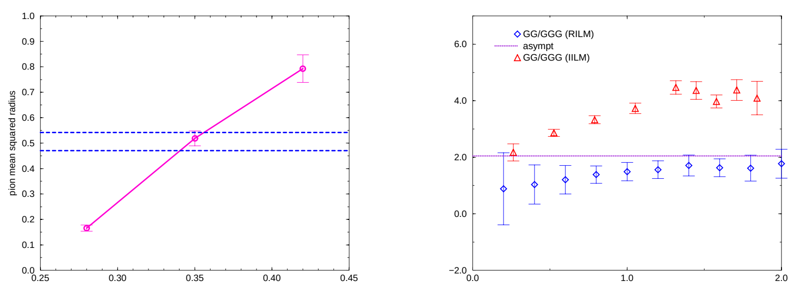

Figure 4: The pion squared radius as a function of the instanton size (left) and (right) the ratio $\Delta\Sigma/\Delta c$ for the random and interacting ensemble as a function of $x = 2y$.

This means that first the charm contribution is opposite to the contribution from the light quarks, in qualitative agreement with lattice QCD calculations for the disconnected light flavour contribution [19] to $g_A^{(0)}$. Though the charm contribution does not fully explain the proton spin puzzle, its magnitude, which is only 3-6 times smaller than the sea contribution of the light quarks, should be measureable in the future by the COMPASS [20] experiment at CERN via open charm production.

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